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Classification of Lepton Currents.

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Summary. — We use a description where the neutrino is four-component, the positive muon, the negative electron, and the neutrino are called leptons, and leptons are conserved. We perform unitary transformations (SU_3) on the three variables μ, ν, e to classify particles and currents. We are led to independent sets of currents which have either a definite parity character or a definite chiral character. Those with definite chiral character are physically acceptable and imply that the positive helicity leptons transform under SU_3 contragrediently with respect to the negative helicity leptons. The transformation properties allow us to assign quantum numbers similar to strangeness and isotopic spin. A baryon-lepton symmetry is formulated implying that positive helicity leptons correspond to p, n, and Z^- (baryon with $S=-3$) and negative helicity leptons correspond to X^+ (baryon with $S=1$), Ξ^0 and Ξ^-. The weak four-lepton Lagrangian is written in a form invariant under lepton isospin transformations (SU_3) and physical consequences are discussed.

Introduction.

The high energy neutrino experiments carried out at Brookhaven [1] have indicated that two distinct two-component neutrinos exist, one coupled to the muon, the other to the electron. Furthermore the absence of $\mu \to e+\gamma$ [2],

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(1) G. DAMBY, J. M. GAILLARD, K. GOULIANOS, L. M. LEDERMANN, N. B. MISTRY, M. SCHWARTZ and J. STEINBERGER: *Proc. 1962 International Conference on High-Energy Physics at CERN*, edited by J. PRENTKI (Geneva, 1962).

(2) D. BARTLET, S. DEVONS and A. M. SACHS: *Phys. Rev. Lett.*, **8**, 120 (1962); A. FRANKEL, J. HALPERN, L. HOLLOWAY, W. WALES, M. YARIAN, O. CHAMBERLAIN, S. CEMONICK and F. PIPKIN: *Phys. Rev. Lett.*, **8**, 123 (1962).

$\mu^- +$ nucleus $\to e^- +$ nucleus ([3]) and $\mu \to 3e$ ([4]) have been established to a high degree of accuracy.

A scheme essentially equivalent to the description with two distinct two-component neutrinos $\nu^{(e)}$ and $\nu^{(\mu)}$, both left-handed, can be obtained using a single four-component neutrino, and the definitions

$$\nu^e = \tfrac{1}{2}(1+\gamma_5)\nu\,,$$

$$\nu^\mu = \tfrac{1}{2}(1+\gamma_5)\nu^c\,,$$

where ν^c is the spinor charge conjugate to ν. One defines as leptons the positive muon, the negative electron and the neutrino. Lepton conservation then forbids all the processes that imply a $\mu \to e$ transition. It also forbids muonium-antimuonium conversion ([5]) and $e^- + e^- \to \mu^- + \mu^-$ ([6]). The charged lepton current takes the form

$$-\frac{i}{2}(\pm\,\bar{e}\gamma a\nu - \bar{\mu}^c\gamma a\nu^c) = -\frac{i}{2}(\pm\,\bar{e}\gamma a\nu + \bar{\nu}\gamma\bar{a}\mu)\,,$$

with $a = \frac{1}{2}(1+\gamma_5)$, and gives consistent explanation of μ-decay and neutrino absorption.

We are thus led to a theory with three basic leptons: the positive muon, the neutrino, and the negative electron; and their respective antileptons. It is then natural to examine the transformation properties of the theory under the group of unitary transformations on three variables, and under its subgroups.

The group U_3 of unitary transformations on three variables can be decomposed as $U_3 = U_1 \times SU_3$ where U_1 is represented by a phase transformation that can be identified with the lepton phase transformation. A subgroup SU_2 of SU_3 is isomorphic to isospin rotations in lepton space and allows for a classification employing isospin and strangeness.

The possible sets of currents are given in Table I. They divide into two groups. The sets of the first group consist of currents that have a definite

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([3]) M. Conversi, L. di Lella, G. Penso, M. Toller and C. Rubbia: *Phys. Rev. Lett.*, **8**, 125 (1962); R. D. Sard, K. M. Crowe and H. Kruger: *Phys. Rev.*, **121**, 619 (1961).

([4]) S. Parker and S. Penman: *Nuovo Cimento*, **23**, 485 (1962); A. I. Alikhanov, A. I. Babaev, M. Ya. Balats, V. S. Kaftanov, L. G. Landsberg, V. A. Lyubimov Yu. V. Obukhov: in *Proc. 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (Geneva, 1962).

([5]) G. Feinberg and S. Weinberg: *Phys. Rev.*, **123**, 1439 (1961); L. Okun and B. Pontecorvo: *Žurn. Èksp. Teor. Fiz.*, **41**, 989 (1961); N. Cabibbo and R. Gatto: *Nuovo Cimento*, **19**, 612 (1961); S. Glashow: *Nuovo Cimento*, **20**, 591 (1961).

([6]) N. Cabibbo and R. Gatto: *Nuovo Cimento*, **19**, 612 (1961).

TABLE I.

Current	Case: (I) $\Lambda_i^{(+)}$ and $\Lambda_i^{(-)}$ are equivalent sets				Case (II): $\Lambda_i^{(+)}$ and $\Lambda_i^{(-)}$ are inequivalent sets			
	$\varrho=1$, $\sigma=1$,	$\varrho=1$, $\sigma=-1$	$\varrho=-1$, $\sigma=1$	$\varrho=-1$, $\sigma=-1$	$\varrho=1,\ \sigma=1$	$\varrho=1,\ \sigma=-1$	$\varrho=-1,\ \sigma=1$	$\varrho=-1,\ \sigma=-1$
$\frac{1}{2}(j_1+ij_2)$	$-\frac{i}{2}\bar{\mu}\gamma\nu$	$-\frac{i}{2}\bar{\mu}\gamma\nu$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5\nu$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5\nu$	$-\frac{i}{2}(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma ae+\bar{\mu}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma ae+\bar{\mu}\gamma a\nu)$
$\frac{1}{2}(j_1-ij_2)$	$-\frac{i}{2}\bar{\nu}\gamma\mu$	$-\frac{i}{2}\bar{\nu}\gamma\mu$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5\mu$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5\mu$	$-\frac{i}{2}(\bar{e}\gamma a\nu+\bar{\nu}\gamma\bar{a}\mu)$	$-\frac{i}{2}(\bar{e}\gamma a\nu+\bar{\nu}\gamma\bar{a}\mu)$	$-\frac{i}{2}(-\bar{e}\gamma a\nu+\bar{\nu}\gamma\bar{a}\mu)$	$-\frac{i}{2}(-\bar{e}\gamma a\nu+\bar{\nu}\gamma\bar{a}\mu)$
j_3	$\frac{i}{2}(\bar{\mu}\gamma\mu-\bar{\nu}\gamma\nu)$				$-\frac{i}{2}(\bar{\nu}\gamma a\nu-\bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu-\bar{\nu}\gamma\bar{a}\nu)$			
$\frac{1}{2}(j_4+ij_5)$	$-\frac{i}{2}\bar{\mu}\gamma e$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5 e$	$\frac{i}{2}\bar{\mu}\gamma\gamma_5 e$	$-\frac{i}{2}\bar{\mu}\gamma e$	$-\frac{i}{2}(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma ae+\bar{\mu}\gamma\bar{a}e)$
$\frac{1}{2}(j_4-ij_5)$	$-\frac{i}{2}\bar{e}\gamma\mu$	$\frac{i}{2}\bar{e}\gamma\gamma_5\mu$	$\frac{i}{2}\bar{e}\gamma\gamma_5\mu$	$-\frac{i}{2}\bar{e}\gamma\mu$	$-\frac{i}{2}(-\bar{e}\gamma a\mu+\bar{e}\gamma\bar{a}\mu)$	$-\frac{i}{2}(\bar{e}\gamma a\mu+\bar{e}\gamma\bar{a}\mu)$	$-\frac{i}{2}(\bar{e}\gamma a\mu+\bar{e}\gamma\bar{a}\mu)$	$-\frac{i}{2}(-\bar{e}\gamma a\mu+\bar{e}\gamma\bar{a}\mu)$
$\frac{1}{2}(j_6+ij_7)$	$-\frac{i}{2}\bar{\nu}\gamma e$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5 e$	$-\frac{i}{2}\bar{\nu}\gamma e$	$\frac{i}{2}\bar{\nu}\gamma\gamma_5 e$	$-\frac{i}{2}(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$	$-\frac{i}{2}(-\bar{\mu}\gamma a\nu+\bar{\nu}\gamma\bar{a}e)$
$\frac{1}{2}(j_6-ij_7)$	$-\frac{i}{2}\bar{e}\gamma\nu$	$\frac{i}{2}\bar{e}\gamma\gamma_5\nu$	$-\frac{i}{2}\bar{e}\gamma\nu$	$\frac{i}{2}\bar{e}\gamma\gamma_5\bar{\nu}$	$-\frac{i}{2}(\bar{\nu}\gamma a\mu+\bar{e}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma a\mu+\bar{e}\gamma\bar{a}\nu)$	$-\frac{i}{2}(\bar{\nu}\gamma a\mu+\bar{e}\gamma\bar{a}\nu)$	$-\frac{i}{2}(-\bar{\nu}\gamma a\mu+\bar{e}\gamma\bar{a}\mu)$
j_8	$-\frac{i}{2}\frac{1}{\sqrt{3}}(\bar{\mu}\gamma\mu+\bar{\nu}\gamma\nu-2\bar{e}\gamma e)$				$-\frac{i}{2}\frac{1}{\sqrt{3}}(2\bar{\mu}\gamma a\mu-\bar{\nu}\gamma a\nu-\bar{e}\gamma ae+\bar{\mu}\gamma\bar{a}\mu+\bar{\nu}\gamma\bar{a}\nu-2\bar{e}\gamma\bar{a}e)$			

behaviour under space inversion. By suitable definition of intrinsic parities sets of the first group consist of currents that transform like vectors. Thus one can exclude such sets, on the basis of parity nonconservation in muon decay. The sets of the second group consist of currents with a definite chiral character. The charged current $\frac{1}{2}(j_1 \pm i j_2)$ coincide, for the sets of the second group, with the established charged lepton currents. This fact allows us to identify that SU_2 subgroup, most suitable for a classification of the currents, as that one whose generators are obtained by integrating the fourth components of $\frac{1}{2}(j_1 \pm i j_2)$ and j_3 over all space.

The basis for the distinction of the current sets into two groups is the following: sets of the first group are obtained if one assumes that the positive-helicity leptons (that we call: μ_+, ν_+, and e_+) transform contragradiently with respect to the negative-helicity leptons (that we call: μ_-, ν_- and e_-); sets of the second group are obtained if the positive-helicity leptons transform contragradiently with the negative-helicity leptons.

In group-theoretical language the distinction is: for sets of the first group both negative-helicity leptons and positive-helicity leptons can be taken to transform according to $D^3(1, 0)$; for sets of the second group the positive-helicity leptons can be taken to transform according to $D^3(1, 0)$, and the negative-helicity leptons will then transform according to $D^3(0, 1)$. $D^3(1, 0)$ and $D^3(0, 1)$ are the two inequivalent three-dimensional representations of SU_3.

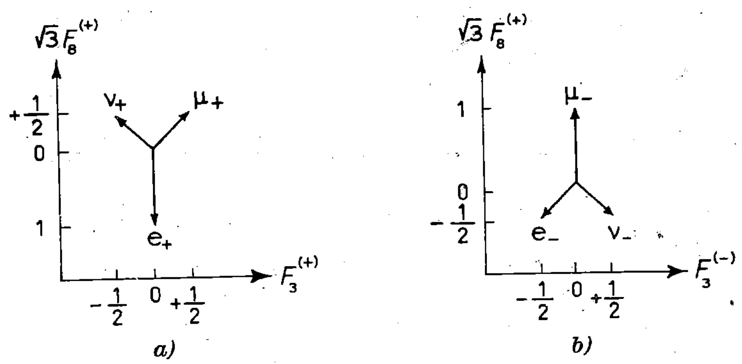

Fig. 1. – *a*) Weight diagram for the positive-helicity leptons; *b*) weight diagram for the negative-helicity leptons.

In Fig. 1*a* and 1*b* we have reported in diagrams the quantum-number assignments for the positive- and for the negative-helicity leptons. The quantum numbers are $F_3^{(+)}$ and $F_8^{(+)}$ for the positive-helicity leptons, and $F_3^{(-)}$ and $F_8^{(-)}$ for the negative-helicity leptons. $F_3^{(\pm)}$ and $F_8^{(\pm)}$ are the space integrals of the currents $j_3^{(\pm)}$ and $j_8^{(\pm)}$, obtained by decomposing each current into a contribution from positive helicities and a contribution from negative helicities. The diagrams of Fig. 1*a* and 1*b* are essentially the weight diagrams for the representations $D^3(1, 0)$ and $D^3(0, 1)$ of SU_3.

We shall introduce quantum numbers $I_3^{(+)} = F_3^{(+)}$ and $S^{(+)} = 2\sqrt{3}\,F_8^{(+)} - L$ (L is the lepton number), and similarly $I_{(3)}^{(-)}$ and $S^{(-)}$. The charge Q is then given by $Q = I_3^{(+)} + \frac{1}{2}(L + S^{(+)}) = I_3^{(-)} + \frac{1}{2}(L + S^{(-)})$, as for strong interacting particles. The particles and currents can then be classified as in Tables II*a*, II*b* and III.

The above classification allows us to establish a correspondence of leptons and lepton currents with baryons and mesons, respectively, that have corresponding quantum numbers (in the sense: lepton number$\leftrightarrow$nucleon number; $Q \leftrightarrow Q$; $S^{(+)}$, $S^{(-)} \leftrightarrow S$; $\boldsymbol{I}^{(+)}$, $\boldsymbol{I}^{(-)} \leftrightarrow \boldsymbol{I}$). In Tables II*a*, II*b* and III we have reported in the last column the « corresponding baryon » for each lepton and the

TABLE II*a*. – *Quantum number assignments to the leptons with positive helicity.*

	Lepton number L	Charge Q	R-strangeness $S^{(+)}$	R-isospin $\lvert \boldsymbol{I}^{(+)} \rvert$	$I_3^{(+)}$	Corresponding baryon
μ_+	$+1$	$+$	0	$\frac{1}{2}$	$\frac{1}{2}$	p
ν_+	$+1$	0	0		$-\frac{1}{2}$	n
e_+	$+1$	$-$	-3	0	0	Z^-

The corresponding barions are indicated in last column. The baryon Z^- has not yet been discovered.

TABLE II*b*. – *Quantum number assignments to the leptons with negative helicity.*

	Lepton number L	Charge Q	L-strangeness $S^{(-)}$	L-isospin $\lvert \boldsymbol{I}^{(-)} \rvert$	$I_3^{(-)}$	Corresponding baryon
μ_-	$+1$	$+$	1	0	0	X^+
ν_-	$+1$	0	-2	$\frac{1}{2}$	$\frac{1}{2}$	Ξ^0
e_-	$+1$	$-$	-2		$-\frac{1}{2}$	Ξ^-

The corresponding baryons are indicated in the last column. The baryon X^+ has not yet been discovered.

« corresponding meson » for each current. The baryons Z^- ($I = 0$, $S = -3$) and X^+ ($I = 0$, $S = +1$), and the mesons $\varphi(I = \frac{1}{2}, S = \pm 3)$ have not been discovered so far.

The correspondence defined in Tables II*a* and II*b* is the correct expression of the so-called baryon-lepton symmetry that was first discussed by GAMBA, OKUBO and MARSHAK [7].

The currents of Table I, by self-coupling, generate a weak four-lepton Lagrangian. For suitable choice of the coefficients such Lagrangian may be invariant under the full unitary group. We can exclude such a possibility directly, on the basis that the resulting theory would be parity-conserving and would not apply to weak interactions.

If we assume invariance under the SU_3 (lepton-isospin) subgroup we are led to the following weak four-lepton Lagrangian:

$$L' = gL_1 + fL_2 + hL_3 ,$$

TABLE III. – *Quantum number assignments to the currents.*

Current	Lepton number L	Charge Q	$S^{(-)}=S^{(+)}$	$\lvert I^{(-)}\rvert = \lvert I^{(+)}\rvert$	$I_3^{(-)}=I_3^{(+)}$	Corresponding mesons
$\frac{1}{2}(j_1+ij_2)$	0	+1	0	1	+1	ρ^+
$\frac{1}{2}(j_1-ij_2)$	0	−1	0	1	−1	ρ^-
j_3	0	0	0	1	0	ρ^0
$\frac{1}{2}(j_4+ij_5)$	0	+2	+3	$\frac{1}{2}$	$\frac{1}{2}$	φ^{++}
$\frac{1}{2}(j_4-ij_5)$	0	−2	−3	$\frac{1}{2}$	$-\frac{1}{2}$	φ^{--}
$\frac{1}{2}(j_6+ij_7)$	0	+1	+3	$\frac{1}{2}$	$-\frac{1}{2}$	φ^+
$\frac{1}{2}(j_6-ij_7)$	0	−1	−3	$\frac{1}{2}$	$\frac{1}{2}$	φ^-
j_8	0	0	0	0	0	ω^0

The corresponding (vector) mesons are indicated in the last column. The mesons φ have not yet been discovered.

where L_1, L_2, and L_3 stay invariant under SU_2: L_1 is the self-coupling of j_1, j_2 and j_3; L the self-coupling of j_4, j_5, j_6, j_7; and L_3 the self-coupling of j_8. The unitary limit obtains when the coupling constants g, f, h are all equal. It is possible to give a stringent upper limit to the ratio f/g. In fact the measured value of the ξ parameter in μ-decay can be shown to imply $f/g < 0.2$.

The coupling to strong currents of j_6 and j_7 would independently lead to

(7) A. GAMBA, R. E. MARSHAK and S. OKUBO: *Proc. Nat. Acad. Sci.*, **45**, 881 (1959).

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difficulties (neutrino flip) [8]. Furthermore if L_2 were generated by intermediate vector mesons it would require double-charged mesons.

It is an important experimental point to check if invariance under SU_2 is satisfied. This can be checked by measuring the scattering of electron-neutrinos by electrons.

Invariance under SU_2 would also imply scattering of muon-neutrinos by electrons, and weak interaction effects in very high energy electron-positron electromagnetic effects. All these effects are difficult to test experimentally but their knowledge seems very important for an understanding of weak couplings.

It is important to note that, whereas in a theory with one two-component neutrino invariance under SU_2 (in the sense used here) implies the absence of neutrino scattering on electron and on muon (in V.A. theory), such processes will take place if the muon-neutrino is different from the electron-neutrino — both neutrinos are scattered by the electron, or by the muon. The origin of the absence of scattering of neutrino on electron and on muon in the one-neutrino theory is in fact a fortuitous cancellation between two amplitudes that only persists as long as the electron-neutrino coincides with the muon-neutrino.

1. – The symmetry group.

1'1. – The 3×3 unit matrix together with a set of 3×3 independent traceless hermitian matrices $\lambda_1, \lambda_2, \ldots, \lambda_8$ generate unitary transformations on three variables. A typical set of λ_i $(i=1, 2, 3, \ldots, 8)$ is that chosen by GELL-MANN [9].

$$(1)\qquad \left\{\begin{aligned}
&\lambda_1=\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}\quad \lambda_2=\begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix}\quad \lambda_3=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&0\end{bmatrix}\\
&\lambda_4=\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix}\quad \lambda_5=\begin{bmatrix}0&0&-i\\0&0&0\\i&0&0\end{bmatrix}\quad \lambda_6=\begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}\\
&\lambda_7=\begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix}\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix}.
\end{aligned}\right.$$

(8) G. FEINBERG, F. GURSEY and A. PAIS: *Phys. Rev. Lett.*, **7**, 208 (1961); S. BLUDMAN: *Phys. Rev.*, **124**, 947 (1961).

(9) M. GELL-MANN: *Phys. Rev.*, **125**, 1067 (1962).

The matrices λ_i satisfy the commutation relations

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k \,, \tag{2}$$

where f_{ijk} is real and totally antisymmetric. The nonzero matrix elements of f_{ijk} are [9]

$$f_{123} = 1 ; \quad f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \tfrac{1}{2} ; \quad f_{458} = f_{678} = \frac{\sqrt{3}}{2} \,. \tag{3}$$

The commutation relations (2) are, of course, still satisfied if, instead of the above set of matrices λ_i, one chooses a new set λ_i' obtained from λ_i by a similarity transformation

$$\lambda_i' = \omega\lambda_i\omega^{-1} \,, \tag{4}$$

where ω is a nonsingular 3×3 matrix.

It is easy to see, however, that the set of 3×3 matrices $\tilde{\lambda}_i$ ($i=1, 2, ..., 8$) given by

$$\tilde{\lambda}_1 = -\lambda_1, \; \tilde{\lambda}_2 = \lambda_2, \; \tilde{\lambda}_3 = -\lambda_3, \; \tilde{\lambda}_4 = -\lambda_4, \; \tilde{\lambda}_5 = \lambda_5, \; \tilde{\lambda}_6 = -\lambda_6, \; \tilde{\lambda}_7 = \lambda_7, \; \tilde{\lambda}_8 = -\lambda_8, \tag{5}$$

still satisfies the commutation relations (2) and cannot be obtained from the λ_i by a similarity transformation of the form (4).

To verify that (5) satisfies the commutation relations (2) it is sufficient to note that any nonzero f_{ijk}, as given by (3), contains the indices 2, 5, 7 an odd number of times, so that any commutation relation is left unchanged if we reverse the sign of all λ_i with i different from 2, 5, 7. The set $\tilde{\lambda}_i$ cannot be obtained by a similarity transformation since such a transformation would leave the set of eigenvalues unchanged for each transformed matrix, whereas the eigenvalues of $\tilde{\lambda}_8$ are different from those of λ_8.

From the set $\tilde{\lambda}_i$, by similarity transformations, we can obtain new sets

$$\lambda_i' = \omega\tilde{\lambda}_i\omega^{-1} \,, \tag{6}$$

that satisfy the commutation relations (2). Conversely it is true that any set of 3×3 matrices that satisfies (2) can be obtained through a similarity transformation either from the set λ_i or from the set $\tilde{\lambda}_i$.

1'2. – We now look ~~for~~ a general transformation to be imposed on our system of lepton fields, described by a multi-component spinor ψ. The spinor ψ describes the muon, the neutrino, and the electron.

The phase transformation expressing the freedom of the lepton gauge will be represented infinitesimally by

$$\psi \to \left(1 + i\frac{\varepsilon}{2}I\right)\psi\,, \tag{7}$$

where I is the unit 3×3 matrix and ε an infinitesimal parameter. We have chosen the positive muon, the neutrino, and the negative electron to be leptons; their antiparticles are antileptons.

Next to (7) we consider the infinitesimal transformations

$$\psi \to \left(1 + i\sum_{i=1}^{8}\varepsilon_i\frac{\Lambda_i}{2}\right)\psi\,, \tag{8}$$

depending on the eight infinitesimal parameters ε_i $(i = 1, 2, \ldots, 8)$.

From relativistic invariance the 3×3 traceless, Hermitian matrices Λ_i can depend from 1 and γ_5. Introducing

$$a = \tfrac{1}{2}(1+\gamma_5)\,,$$

$$\bar{a} = \tfrac{1}{2}(1-\gamma_5)\,,$$

with the properties $aa = a$, $\bar{a}\bar{a} = \bar{a}$, $a\bar{a} = \bar{a}a = 0$, we write

$$\Lambda_i = \Lambda_i^{(-)}a + \Lambda_i^{(+)}\bar{a}\,. \tag{9}$$

Furthermore

$$[\Lambda_i^{(-)}, \Lambda_j^{(-)}] = 2if_{ijk}\Lambda_k^{(-)}\,, \tag{10}$$

$$[\Lambda_i^{(+)}, \Lambda_j^{(+)}] = 2if_{ijk}\Lambda_k^{(+)}\,. \tag{10'}$$

From (9), (10) and (10′)

$$[\Lambda_i, \Lambda_j] = [\Lambda_i^{(-)}, \Lambda_j^{(-)}]a + [\Lambda_i^{(+)}, \Lambda_j^{(+)}]\bar{a} = 2if_{ijk}\Lambda_k\,. \tag{11}$$

We can now put, without loss of generality,

$$\Lambda_i^{(+)} = \lambda_i\,, \qquad \Lambda_i^{(-)} = \lambda_i'\,, \tag{12}$$

where the λ_i' are either of the form (4) or the form (6).

The transformation matrix ω is supposed to be unitary

$$\omega\omega^+ = 1 \tag{13}$$

to preserve the Hermitian character of the infinitesimal generators Λ_i.

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Associated to the transformation (8) are currents

$$j_i^\mu = -\frac{i}{2}\bar{\psi}\gamma^\mu \Lambda_i \psi = j_i^{(-)\mu} + j_i^{(+)\mu}, \tag{14}$$

$$j_i^{(-)\mu} = -\frac{i}{2}\bar{\psi}\gamma^\mu a \Lambda_i^{(-)}\psi, \tag{15}$$

$$j_i^{(+)\mu} = -\frac{i}{2}\bar{\psi}\gamma^\mu \bar{a} \Lambda_i^{(+)}\psi. \tag{16}$$

For the operators, that we shall call generators,

$$F_i = -i\int j_i^\mu \,\mathrm{d}\sigma_\mu = F_i^{(-)} + F_i^{(+)}. \tag{17}$$

The equal-time commutation relations can be derived from (11)

$$[F_i^{(-)}, F_j^{(-)}] = i f_{ijk} F_k^{(-)}, \quad [F_i^{(+)}, F_j^{(+)}] = i f_{ijk} F_k^{(+)}. \tag{18}$$

Also we recall that the currents (14) are not all conserved. Their divergences are given by

$$\frac{\partial j_i^\mu}{\partial x^\mu} = \frac{\partial L}{\partial \varepsilon_i}, \tag{19}$$

where L is that part of the Lagrangian that does not stay invariant under the transformation (8).

2. – Costruction of the generators.

2·1. – The choice of the set λ_i' in (12) is severely limited by the conservation laws. Let us first discuss the implications of charge conservation.

The matrix q representing the charge Q is defined by the eigenvalue equations

$$\begin{cases} q\psi(\mu) = \psi(\mu), \\ q\psi(\nu) = 0, \\ q\psi(e) = -\psi(e). \end{cases} \tag{20}$$

In terms of our basic sets (1) and (5)

$$q = \tfrac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8) = -\tfrac{1}{2}(\tilde{\lambda}_3 + \sqrt{3}\tilde{\lambda}_8). \tag{21}$$

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We also note that

$$\mathrm{Tr}\,[q] = 0 \,. \tag{22}$$

The equal-time commutation relations of the charge operator Q with the operators $F_i^{(-)}$ and $F_i^{(+)}$ will be of the kind

$$\begin{cases} [Q,\, F_i^{(+)}] = C_{ik} F_k^{(+)} \,, \\ [Q_i^{(-)},\, F_i^{(-)}] = C_{ik} F_k^{(-)} . \end{cases} \tag{23}$$

The matrix elements C_{ik} are given by

$$[q,\, \lambda_i] = C_{ik}\, \lambda_k \tag{24}$$

and we find that

$$[q,\, \lambda_i'] = C_{ik}\, \lambda_k' \tag{25}$$

must also be valid.

2'2. – We discuss first case (I): λ_i' is of the form (4), *i.e.* the set λ_i' is equivalent to the basic set λ_i given in (1). From (4) and (25) we obtain.

$$[q,\, \omega\lambda_i\omega^{-1}] = C_{ik}\, \omega\lambda_k\, \omega^{-1}$$

or

$$[q',\, \lambda_i] = C_{ik}\, \lambda_k \,, \tag{26}$$

where we have defined

$$q' = \omega^{-1} q \omega \,. \tag{27}$$

From (24) and (27) we find

$$[q - q',\, \lambda_i] = 0 \,. \tag{28}$$

Furthermore from (27)

$$\mathrm{Tr}\,[q'] = \mathrm{Tr}\,[q] = 0 \,.$$

Thus (28) implies

$$[q,\, \omega] = 0 \,. \tag{29}$$

Equation (29) is a very stringent condition on ω. In the chosen representation q is a diagonal matrix. Furthermore its eigenvalues are all different. It follows that ω must be diagonal in the same representation. We also recall that ω is unitary. The similarity transformation (4) can thus be expressed in terms of two independent real phases that we call α and β.

To exhibit explicitly the transformation (4) from the sets λ_i to the set λ'_i we define a matrix $W^{(\mathrm{I})}$ such that

$$\lambda'_i = W^{(\mathrm{I})}_{ik}\lambda_k\,. \tag{30}$$

By performing the transformation (4) with a unitary and diagonal ω we find

$$W^{(\mathrm{I})} = \begin{pmatrix} R(\alpha) & & & & \\ & 1 & & & \\ & & R(\alpha)R(\beta) & & \\ & & & R(\beta) & \\ & & & & 1 \end{pmatrix}, \tag{31}$$

where $R(\gamma)$ represents a two-dimensional rotation of an angle γ

$$R(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}. \tag{32}$$

2·3. – Next we discuss case (II): λ'_i is of the form (6), *i.e.* the set λ'_i is equivalent to the basic set $\tilde{\lambda}_i$ given by (5). We must satisfy

$$[q, \lambda'_i] = C_{ik}\lambda'_k\,. \tag{33}$$

Using eq. (6)

$$[q, \omega\tilde{\lambda}_i\omega^{-1}] = C_{ik}\omega\tilde{\lambda}_k\omega^{-1}$$

or

$$[q', \tilde{\lambda}_i] = C_{ik}\tilde{\lambda}_k\,, \tag{34}$$

with q' defined as in (27). We compare (34) with

$$[q, \tilde{\lambda}_i] = -\,C_{ik}\tilde{\lambda}_k \tag{35}$$

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that follows from (21). We obtain

$$[q + q', \tilde{\lambda}_i] = 0 \tag{36}$$

and, again recalling that $\mathrm{Tr}[q'] = \mathrm{Tr}[q] = 0$, we derive the anticommutation relation

$$\{q, \omega\} = 0\,. \tag{37}$$

Equation (37) (compare with (29), for case I) strongly limits the form of ω. In the chosen representation q is diagonal and its eigenvalues are ordinately $+1$, 0, -1. In the same representation ω must therefore be antidiagonal. Recalling again that ω must be unitary we find that the similarity transformation (6) can be expressed in terms of two independent real phases θ and φ.

We define a matrix $W^{(\mathrm{II})}$ such that

$$\lambda'_i = W^{(\mathrm{II})}_{ik}\lambda_k\,. \tag{38}$$

We find, by performing the transformation (6) with an antidiagonal and unitary ω, and using (5), that $W^{(\mathrm{II})}$ is given by

$$W^{(\mathrm{II})} = \left[\begin{array}{c|c|c|c|c} & & & R(\theta) & \\ \hline & -\frac{1}{2} & & & \frac{\sqrt{3}}{2} \\ \hline & & -R(\theta)R(\varphi) & & \\ \hline R(\varphi) & & & & \\ \hline & \frac{\sqrt{3}}{2} & & & \frac{1}{2} \end{array}\right], \tag{39}$$

with $R(\omega)$ given by (32).

2·4. – The above construction gives all the sets λ'_i that are consistent with the commutation relations (23). They are obtained by using (30) and (31) or (38) and (39).

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The theories that one constructs by such a procedure do not in general satisfy the requirement of invariance under time reversal. Though it might perhaps be of interest to examine closer such theories, we shall in the following impose a reality condition that guarantees invariance under time reversal.

Namely, we shall require that, if $\lambda_i^* = \eta_i \lambda_i$, where $\eta_i = \pm 1$ (as can be seen from (1)), also $\lambda_i^{*} = \eta_i \lambda_i$. The condition implies that $\omega_{ik}(\eta_i - \eta_k) = 0$ for any pair i, k, and for both $W^{(\mathrm{I})}$ and $W^{(\mathrm{II})}$. It is equivalent to requiring that the phases α, β and φ must all be multiples of π. In this way we obtain four possible sets of λ_i' in case (I) and four possible choices in case (II). If we introduce symbols ϱ and σ that can independently take values ± 1, we find in case (I)

$$\lambda_1' = \varrho\lambda_1, \quad \lambda_2' = \varrho\lambda_2, \quad \lambda_3' = \lambda_3, \quad \lambda_4' = \varrho\sigma\lambda_4, \quad \lambda_5' = \varrho\sigma\lambda_5, \quad \lambda_6' = \sigma\lambda_6, \quad \lambda_7' = \sigma\lambda_7, \quad \lambda_8' = \lambda_8, \tag{40}$$

and in case (II)

$$\begin{cases} \lambda_1' = \varrho\lambda_6, \quad \lambda_2' = \varrho\lambda_7, \quad \lambda_3' = -\dfrac{1}{2}\lambda_3 + \dfrac{\sqrt{3}}{2}\lambda_8, \quad \lambda_4' = -\varrho\sigma\lambda_4, \\ \lambda_5' = -\varrho\sigma\lambda_5, \quad \lambda_6' = \sigma\lambda_1, \quad \lambda_7' = \sigma\lambda_2, \quad \lambda_8' = \dfrac{\sqrt{3}}{2}\lambda_3 + \dfrac{1}{2}\lambda_8 . \end{cases} \tag{41}$$

It is easy to verify directly that the commutation rules (2) are verified by the λ_i' both in case (I) and in case (II). In case (I) they are obviously verified for $\varrho = \sigma = +1$, and each nonzero f_{ijk} in (3) contains an even number of time indices of λ's that are multiplied by ϱ in (40), and an even number of time indices of λ's that are multiplied by σ. Similarly for case (II) the commutation relations can be verified to hold for $\varrho = \sigma = 1$, and then found to hold for the other choices.

3. – The currents.

We recall that the currents j_i are given by (14) with the identification (12) of the matrices Λ_i. The possible sets of currents obtained for case (I) and for case (II) are reported in Table I.

We see that we are thus led two different kinds of theories:

Case (I): The left-handed $\Lambda_i^{(-)}$ and right-handed $\Lambda_i^{(+)}$ generators are equivalent sets. (In group-theoretical language one would say that they belong to the same three-dimensional representation of SU_3, the special unitary group in three dimensions.) In this case one is led to parity-conserving theories, as shown in the first four columns of Table I. For $\varrho = 1$, $\sigma = 1$ one would formally assign same parity to μ, ν, e; for $\varrho = 1$, $\sigma = -1$ one would formally

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assign same parity to μ and ν and opposite to e; for $\varrho=-1$, $\sigma=1$ one would formally assign same parity to μ and e and opposite to ν. Of course the lepton number and charge superselection rules would make such parity assignments of no particular physical content.

Case (II). The left-handed, $\Lambda_i^{(-)}$, and right-handed, $\Lambda_i^{(+)}$, generators are inequivalent sets (in group-theoretical language they belong to different three-dimensional representations of SU_3). In this case one is led to theories of the $A \pm V$ form.

All experiments on weak leptonic processes (including high-energy neutrino absorption) are consistent with a charged leptonic current of the form

$$-\frac{i}{2}(\pm \bar{\nu}\gamma a e + \bar{\mu}\gamma \bar{a}\nu) . \tag{42}$$

We see that (43) is exactly the form that $\frac{1}{2}(j_1 + ij_2)$ takes in case (II). The current classification suggested in case (II) seems therefore to be particularly convenient as a possible framework for discussing leptonic processes.

We see that essentially two kinds of theories have emerged:

Case (I) parity-conserving theories.

Case (II) chiral theories.

No assumptions about parity conservation or chirality had been made at the start. The main assumption was the specific form of the commutation relation (11).

From now on we shall only consider case (II) as case (I) does not apply to parity nonconserving leptonic processes. From j_3 and j_8 we can obtain two currents

$$-\frac{i}{2}(j_3 + \sqrt{3}j_8) = -i(\bar{e}\gamma e + \bar{\mu}\gamma\mu) , \tag{43}$$

and

$$-\frac{i}{2}(3j_3 - \sqrt{3}j_8) = -i(2\bar{\nu}\gamma\gamma_5\nu - \bar{\mu}\gamma\gamma_5\mu - \bar{e}\gamma\gamma_5 e) . \tag{44}$$

The current (43) is the electromagnetic current, exactly conserved; the current (44) is an axial current and its rigourous conservation is broken by the presence of the mass terms.

4. – Physical interpretation. The baryon-lepton symmetry.

4·1. – We shall examine more closely the physical interpretation of case (II). The transformation (8) when Λ_i is of the form (9) and $\Lambda_i^{(-)}$ and $\Lambda_i^{(+)}$ are inequivalent implies that the component ψ_- of ψ, with negative helicity, transform differently (contragrediently) from the component ψ_+, with positive helicity. In group-theoretical language [10] one would say that ψ_+ (describing μ_+, ν_+, e_+) transforms according to the representation $D^3(1, 0)$ of the unitary unimodular group SU_3, whereas ψ_- (describing μ_-, ν_-, e_-) transforms according to $D^3(0, 1)$.

From the commutation relations (18) one sees that F_1 and F_8 can be diagonalized simultaneously, for each set of right and left generators. We can thus choose $F_3^{(+)}$ and $\sqrt{3}F_8^{(+)}$ as quantum numbers to label the one-particle states of positive helicity, and $F_3^{(-)}$ and $\sqrt{3}F_8^{(-)}$ to label the one-particle states of negative helicity. We report the eigenvalues of F_3 and of $\sqrt{3}F_8$ on orthogonal axes and we get the diagrams of Fig. 1*a* and 1*b*.

The diagrams refer both to leptons (as opposite of antileptons): the charge states are, in both diagrams, μ^+, ν, e^-. In group theory one calls diagrams similar to those in Fig. 1*a* and 1*b* « weight diagrams ». The diagram 1*a* is the weight diagram for the representation $D^3(1, 0)$, and that in 1*b* is the weight diagram for the representation $D^3(0, 1)$.

The charge Q is given by

$$Q = F_3^{(+)} + \sqrt{3}F_8^{(+)} \tag{45}$$

for the positive-helicity particles, and by

$$Q = F_3^{(-)} + \sqrt{3}F_8^{(-)} \tag{46}$$

for the negative-helicity particles. One can verity that it is $+1$, 0, -1 for μ, ν, e, independently of their helicities.

4·2. – It is suggestive to introduce quantum numbers that are analogous to the third component of isotopic spin I_3, and to the strangeness S. Their definition is uniquely provided by comparing (45) and (46) with the usual relation

$$Q = I_3 + \frac{N + S}{2},$$

for strong interacting particles, and by comparing with the weight diagrams of Fig. 1*a* and Fig. 1*b*. We find (L is the lepton number)

$$F_3^{(+)} = I_3^{(+)}, \qquad \sqrt{3}F_8^{(+)} = \tfrac{1}{2}Y^{(+)} = \tfrac{1}{2}(L + S^{(+)}), \tag{47}$$

$$F_3^{(-)} = I_3^{(-)}, \qquad \sqrt{3}F_8^{(-)} = \tfrac{1}{2}Y^{(-)} = \tfrac{1}{2}(L + S^{(-)}); \tag{48}$$

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(10) R. E. Behrends, J. Dreitlein, C. Fronsdal and W. Lee: *Rev. Mod. Phys.*, **34**, 1 (1962).

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we thus see that μ_+, ν_+ form a doublet, with isotopic-spin $|\boldsymbol{I}^+| = \frac{1}{2}$ and strangeness $S^{(+)} = 0$, and e_+ is a singlet, with isotopic-spin $|\boldsymbol{I}^+| = \frac{1}{2}$ and strangeness $S^{(+)} = -3$; μ_- is a singlet, with isotopic-spin $|\boldsymbol{I}^{(-)}| = 0$ and strangeness $S^{(-)} = 1$, ν_-, e_- form a doublet with isotopic spin $|\boldsymbol{I}^{(-)}| = \frac{1}{2}$ and strangeness $S^{(-)} = -2$. Such a quantum number assignment is summarized in Table II*a* for the positive-helicity leptons and in Table II*b* for the negative-helicity leptons.

In Tables II*a* and II*b* we have indicated in the last column the corresponding baryon *i.e.* with the corresponding quantum numbers. The correspondence between quantum numbers is

$$(49) \qquad L \leftrightarrow N\,; \quad Q \leftrightarrow Q\,; \quad S^{(+)},\, S^{(-)} \leftrightarrow S\,; \quad \boldsymbol{I}^{(+)},\, \boldsymbol{I}^{(-)} \leftrightarrow \boldsymbol{I}\,.$$

The baryon Z^- (a baryon with $I = 0$ and strangeness $S = -3$) has not yet been reported, neither as a stable particle nor as a resonance. Similarly the baryon X^+ (a baryon with $I = 0$, and strangeness $S = +1$) has not yet been reported.

There has been lastly an interest in trying to find a correspondence principle between baryons and leptons. In particular GAMBA, MARSHAK, and OKUBO ([7]) proposed, at the time when one neutrino was known, a correspondence

$$p \leftrightarrow \nu\,, \qquad n \leftrightarrow e^-\,, \qquad \Lambda \leftrightarrow \mu^-\,.$$

After the discovery of the second neutrino there has been some confusion on the subject. Tables II*a* and II*b* indicate how the right correspondence law (also called baryon-lepton symmetry) must be formulated.

We now examine the classification of the currents j_i according to our quantum numbers isospin and strangeness. In group-theoretical language one says that the currents transform according to the regular eight-dimensional representation of SU_3. There is only one eight-dimensional representation of SU_3, called $D^8(1;1)$—in contrast to the existence of the two inequivalent three-dimensional representations $D^3(1, 0)$ and $D^3(0, 1)$.

The currents j_i are, as indicated in eq. (14), the sum of a current constructed from the negative-helicity particles, $j_i^{(-)}$, and a current constructed from the positive-helicity particles, $j_i^{(+)}$. The currents $j_i^{(-)}$ bear quantum numbers $I^{(-)}$ and $S^{(-)}$, while the currents $j_i^{(+)}$ bear quantum numbers $I^{(+)}$ and $S^{(+)}$. However, for the currents, all the $(+)$ quantum numbers coincide with the $(-)$ quantum numbers.

In Table III we summarize the quantum-number assignments for the currents and indicate the boson, corresponding to each current, *i.e.* the (vector) meson having corresponding quantum numbers in the sense of the correspondence (49). The mesons φ (two charge coniugate doublets $\varphi^{++}\varphi^{+}$ and $\varphi^{-}\varphi^{--}$) with $I = \frac{1}{2}$ and strangeness $S = \pm 3$ have not yet been found.

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5. – Weak four-lepton processes.

5·1. – If one assumes that the currents couple to intermediate vector bosons one has to introduce doubly-charged vector bosons coupled to the strangeness ± 3 currents. Such a feature seems to us rather unpleasant. Our point of view will be that, though the scheme summarized in Table III is useful for a classification of the currents, for the assignment of the quantum numbers S and I to the leptons, and to derive a formal baryon-lepton symmetry, the weak coupling does not respect the full unitary symmetry. Such a possibility, though attractive it may be, would be inconsistent with weak interaction data. In particular the $\mu \to e+\nu+\nu$ decay would have to arise from a parity-conserving coupling if one assumes the full unitary symmetry. On the other hand it seems reasonable to assume that the weak coupling is invariant with respect to the subgroup SU_2 of SU_3 whose generators are F_1, F_2, and F_3. Such an assumption leads to the right Hamiltonian for μ-decay and to various predictions for other leptonic processes of more difficult observation. The scheme would thus allow for conservation of $I^{(+)}$, $I^{(-)}$, $S^{(+)}$ and $S^{(-)}$, in the limit of zero lepton masses. Such conservation laws will be violated by the lepton mass terms according to the divergence equations (19).

We now write down the four-lepton weak interaction Lagrangian obtained from the currents of Table I on the assumption of invariance under the SU_2 subgroup of SU_3 with generators F_1, F_2, F_3. This subgroup is the isotopic spin subgroup of the full unitary group. Under such a subgroup μ_+ and ν_+ transform like spinor components among themselves, e_+ remains unchanged, and at the same time, μ_- remains unchanged, and ν_- and e_- transform like spinor components among themselves. Furthermore the currents j_1, j_2, j_3 transform among themselves like vector components, j_8 remains unchanged, $\frac{1}{2}(j_4+ij_5)$, $\frac{1}{2}(j_6+ij_7)$, and $\frac{1}{2}(j_6-ij_7)$, $\frac{1}{2}(j_4-ij_5)$ transform into each other like pairs of charge-conjugate spinors.

We write the four-lepton weak interaction Lagrangian obtained from the currents of Table I in the invariant form (under the SU_2 subgroup)

$$L = gL_1 + fL_2 + hL_3 \,, \tag{50}$$

where L_1, L_2, L_3 are given in Table IV, and g, f, h are coupling constants. The Lagrangian L_1 arises from the invariant self-coupling of the currents j_1, j_2, j_3; the Lagrangian L_2 arises from the invariant self-coupling of the currents j_4, j_5, j_6, j_7; the Lagrangian L_3 is the invariant self-coupling of j_8 to itself. The limit of full unitary symmetry corresponds to

$$g = f = h \,. \tag{51}$$

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TABLE IV. – *Invariant four-fermion Lagrangians.*

Invariant Lagrangians under SU_2	Particles coupled
$L_1 = \varrho(\bar{\nu}\gamma ae)(\bar{\nu}\gamma\bar{a}\mu) + \varrho(\bar{\mu}\gamma\bar{a}\nu)(\bar{e}\gamma a\nu) +$	$\mu e \nu_+ \nu_-$
$+ \frac{1}{2}(\bar{\nu}\gamma\nu)(\bar{e}\gamma ae) + \frac{1}{2}(\bar{\nu}\gamma\nu)(\bar{\mu}\gamma\bar{a}\mu) +$	$ee\nu_+\nu_+$, $ee\nu_-\nu_-$, $\mu\mu\nu_+\nu_+$, $\mu\mu\nu_-\nu_-$
$- \frac{1}{2}(\bar{e}\gamma ae)(\bar{\mu}\gamma\bar{a}\mu) +$	$\mu\mu ee$
$+ \frac{1}{4}(\bar{e}\gamma ae)(\bar{e}\gamma ae) + \frac{1}{4}(\bar{\mu}\gamma\bar{a}\mu)(\bar{\mu}\gamma\bar{a}\mu) + \frac{1}{4}(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu)$	$eeee$, $\mu\mu\mu\mu$, $\nu_+\nu_+\nu_+\nu_+$; $\nu_-\nu_-\nu_-\nu_-$; $\nu_+\nu_+\nu_-\nu_-$
$L_2 = \sigma(\bar{\mu}\gamma a\nu)(\bar{e}\gamma\bar{a}\nu) + \sigma(\bar{\nu}\gamma\bar{a}e)(\bar{\nu}\gamma a\mu) +$	$\mu e\nu_+\nu_-$
$+ [\bar{\mu}\gamma(a - \varrho\sigma\bar{a})e][\bar{e}\gamma(a - \varrho\sigma\bar{a})\mu] +$	$\mu\mu ee$
$+ (\bar{\mu}\gamma a\mu)(\bar{\nu}\gamma a\nu) + (\bar{\nu}\gamma\bar{a}\nu)(\bar{e}\gamma\bar{a}e)$	$\mu\mu\nu_-\nu_-$, $ee\nu_+\nu_+$
$L_3 = \frac{1}{12}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)][2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)] +$	$\mu\mu\mu\mu$
$+ \frac{1}{12}[2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)][2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)] +$	$eeee$
$+ \frac{1}{12}(\bar{\nu}\gamma\gamma_5\nu)(\bar{\nu}\gamma\gamma_5\nu) +$	$\nu_+\nu_+\nu_+\nu_+$, $\nu_-\nu_-\nu_-\nu_-$, $\nu_+\nu_+\nu_-\nu_-$
$+ \frac{1}{6}[2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)](\bar{\nu}\gamma\gamma_5\nu) +$	$ee\nu_+\nu_+$, $ee\nu_-\nu_-$
$- \frac{1}{6}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)](\bar{\nu}\gamma\gamma_5\nu) +$	$\mu\mu\nu_+\nu_+$, $\mu\mu\nu_-\nu_-$
$- \frac{1}{6}[2(\bar{\mu}\gamma a\mu) + (\bar{\mu}\gamma\bar{a}\mu)][2(\bar{e}\gamma\bar{a}e) + (\bar{e}\gamma ae)]$	$\mu\mu ee$

For each term we have indicated on the same row the particles that are coupled. Neutrinos (with positive lepton number) with positive helicity are called ν_+, with negative helicity ν_-. Both ν_+ and ν_- are scattered by electrons. The notation employing two neutrinos is $\nu_-^{(e)} = \nu_-$ and $\nu_-^{(\mu)} = \nu_+^{(c)}$.

As we have already pointed out such limit would lead to a parity conserving L and therefore can be excluded.

5'2. – We shall now use the existing data on muon decay to put a very strong upper limit on the ratio f/g, that strongly suggests that f is indeed zero thus excluding the Lagrangian L_2. As we have said L_2 arises from the self-couplings of the isospin ± 3 currents, according to the classification of Table III. If such a self-coupling originates through intermediate vector bosons it would require doubly-charged bosons. We consider such a feature a pleasant aspect of the absence of L_2.

To derive the mentioned upper limit on f/g let us write down, according to (50) and Table IV, the muon-decay Hamiltonian: there are two contributions, as shown in Table IV, one from L_1 and one from L_2. They add up to give for the muon-decay Hamiltonian

$$H' = \varrho g(\bar{\nu}\gamma ae)(\bar{\nu}\gamma\bar{a}\mu) + \sigma f(\bar{\nu}\gamma\bar{a}e)(\bar{\nu}\gamma a\mu) + \text{h. c.} \tag{52}$$

Using

$$\psi^c = C^{-1}\bar{\psi}\,, \qquad \bar{\psi}^c = C\psi\,,$$

where C is the charge conjugation matrix, satisfying $CC^+ = 1$, $C = -C^T$, and

$C\gamma_\mu C^{-1} = -\gamma_\mu^T$, and the anticommutation of the spinor fields, we rewrite (52) in the form

$$H' = -\varrho g(\bar{\nu}\gamma a e)(\bar{\mu}^c \gamma a \nu^c) - \sigma f(\bar{\nu}\gamma \bar{a} e)(\bar{\mu}^c \gamma \bar{a} \nu^c) + \text{h. c.} \tag{53}$$

We next make use of Fierz's reordering theorem and write

$$H' = -\varrho g(\bar{\nu}\gamma a \nu^c)(\bar{\mu}^c \gamma a e) - \sigma f(\bar{\nu}\gamma \bar{a} \nu^c)(\nu^c \gamma \bar{a} e) + \text{h. c.} \tag{54}$$

Recalling that

$$\bar{\nu}^c \gamma \nu = 0$$

we can finally write for H'

$$4\varrho H' = (\bar{e}\gamma(q + p\gamma_5)\mu^c)(\nu^c \gamma\gamma_5 \nu)\,, \tag{55}$$

where

$$\begin{cases} p = -g - \varrho\sigma f\,, \\ q = -g + \varrho\sigma f\,. \end{cases} \tag{56}$$

The physical consequences of (55) can directly be read off from the paper on the Pauli-Pursey invariants in μ-decay [11] of which we report the relevant conclusion.

All the physical results (in the limit of zero electron mass and without observing the neutrinos) from a general A, V hamiltonian for μ-decay

$$H_{\text{int}} = \sum_{i=A,V} (\bar{e}\Gamma_i \mu^c)\{(\bar{\nu}\Gamma_i(g_i + g_i'\gamma_5)\nu) + (\bar{\nu}^c\Gamma_i(f_i + f_i'\gamma_5)\nu) + (\bar{\nu}\Gamma_i(h_i + h_i'\gamma_5)\nu^c) + \text{h. c.}\}.$$

with real coupling constants, depend from the three invariants

$$\begin{aligned} b &= g_V^2 + g_V'^2 + g_A^2 + g_A'^2 + 2(f_V'^2 + f_A^2 + h_V'^2 + h_A^2)\,, \\ b' &= 2[g_V g_A' + g_V' g_A + 2(f_V' f_A + h_V' h_A)]\,, \\ \beta &= g_V^2 + g_V'^2 - g_A^2 - g_A'^2 + 2(f_V'^2 - f_A^2 + h_V'^2 - h_A^2)\,. \end{aligned}$$

For such A, V hamiltonian, and thus also for (55), the commonly used μ-decay parameters ϱ and δ [12] take the values

$$\varrho = \tfrac{3}{4}\,, \qquad \delta = \tfrac{3}{4}$$

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(11) R. Gatto and G. Lüders: *Nuovo Cimento*, **7**, 806 (1958).

(12) See J. Steinberger: *Rendiconti S.I.F. Corso XI* (Bologna, 1962), p. 375.

independently from the coupling constants. However for the parameter ξ on has

(57) $$\xi = -\frac{b'}{b} = -\frac{2pq}{p^2+q^2}\,.$$

Using the value $\xi = -0.95 \pm 0.5$ taken from reference [12] (this value is computed on the assumption $\delta = \frac{3}{4}$), from (57) and (56) we find

(58) $$f < 0.2g\,.$$

If $f = 0$ the muon decay Hamiltonian (55) takes the simple form

(59) $$H' = (\text{constant})\ (\bar{e}\gamma(1+\gamma_5)\mu^c)(\bar{\nu}^c\gamma\gamma_5\nu)\,.$$

5·3. – We now discuss other physical consequences of the Lagrangian (50). As we have shown that very presumably $f = 0$, we shall confine our discussion to the processes produced by L_1 and L_3. It is impossible at this moment to estimate the ratio h/g, giving the fraction of L_3 that may be present. However all the remarks we shall make apply also to L_1 alone, in which case they only depend from the presence of the self-coupling of the neutral current j_3 in L_1.

Let us first compare our notation with the notation employing $\nu_-^{(e)}$ and $\nu_-^{(\mu)}$ (electron and muon neutrino respectively, both with negative helicity).

To this end we write, from Table I (taking $\varrho = 1$, $\sigma = 1$),

(60) $$\frac{1}{2}(j_1 - ij_2) = -\frac{i}{2}(\bar{e}\gamma a\nu + \bar{\nu}\gamma\bar{a}\mu) = -\frac{i}{2}(\bar{e}\gamma a\nu - \bar{\mu}^c\gamma a\nu^c)\,.$$

Thus the identification is

(61) $$\begin{cases} \nu_-^{(e)} = a\nu\,, \\ \nu_-^{(\mu)} = a\nu^c\,, \end{cases}$$

to compare with our notations

(62) $$\begin{cases} \nu_- = a\nu\,, \\ \nu_+ = \bar{a}\nu\,. \end{cases}$$

We note that

$$a\nu^c = aC^{-1}\bar{\nu} = C^{-1}CaC^{-1}\bar{\nu} = C^{-1}a^T\bar{\nu} = C^{-1}(\bar{\nu}a) = C^{-1}\bar{\nu}_+ = \nu_+^c\,.$$

Substituting into (61) we have

$$\left\{\begin{array}{l}\nu_-^{(e)} = \nu_- ,\\ \nu_-^{(\mu)} = \nu_+^c .\end{array}\right. \tag{63}$$

In L_1 the coupling of the two charged currents $\frac{1}{2}(j_1+ij_2)$ and $\frac{1}{2}(j_1-ij_2)$ gives rise to couplings $\mu e\nu_+\nu_-$ or, in the two-neutrino language, $\mu e\nu^{(e)}\nu^{(\mu)}$.

The same coupling among charged currents gives rise to couplings $ee\nu_-\nu_-$ and $\mu\mu\nu_+\nu_+$ or, in the two-neutrino language, $ee\nu^{(e)}\nu^{(e)}$ and $\mu\mu\nu^{(\mu)}\nu^{(\mu)}$.

However the self-coupling of the neutral current j_3 gives also rise to couplings $ee\nu^{(\mu)}\nu^{(\mu)}$ and $\mu\mu\nu^{(e)}\nu^{(e)}$, as shown in Table IV.

So a possible test of the presence of a self-coupled neutral current could be the following. If the lepton current coupled in the decay of the pion is $\frac{1}{2}(j_1\pm ij_2)$, the neutrinos emitted in $\pi\to\mu\nu$ decay are $\nu^{(\mu)}$, as usually assumed. Such neutrinos should not scatter on electrons if the weak four-lepton coupling is only due to the self-coupling of $\frac{1}{2}(i_1\pm ij_2)$. Any evidence for a $\nu^{(\mu)}$-e cross-section would imply a more complicated coupling, and, perhaps most simply a self-coupling of j_3, as suggested by our invariance under SU_2.

Of course, the postulated additive lepton conservation rule forbids processes such as

$$\mu \to e + \gamma , \tag{64}$$

$$\left\{\begin{array}{l}\mu \to e + e + e ,\\ \mu^- + \text{nucleus} \to e^- + \text{nucleus} .\end{array}\right. \tag{65}$$

As shown in Table IV processes such as

$$e^- + \mu^+ \to \mu^+ + e^- , \tag{66}$$

$$e^- + e^+ \to \mu^+ + \mu^- , \tag{67}$$

are possible, and they are predicted by L_1, arising through the self-coupling of j_3. Observation of (67) with colliding beams would require a very high energy [13].

Processes like

$$e^- + e^- \to \mu^- + \mu^- , \tag{68}$$

$$e^- + \mu^+ \to \mu^- + e^+ , \tag{69}$$

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[13] N. CABIBBO and R. GATTO: *Phys. Rev.*, **124**, 1577 (1961).

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are forbidden by the additive lepton conservation rule assumed here. Reaction (68) could be obtained from colliding electron beams [6], reaction (69) is the so-called muonium-antimonium transition [5]. With the lepton number assignment we have used (μ^+, e^- are leptons), reactions (68) and (69) would require a multiplicative lepton selection rule [6].

RIASSUNTO

In questo lavoro facciamo uso di una descrizione con un solo neutrino a quattro componenti. Il muone positivo, l'elettrone negativo, ed il neutrino sono considerati leptoni e si assume la conservazione dei leptoni. Per classificare le particelle e le correnti si operano trasformazioni unitarie (SU_3) sulle tre variabili μ, ν, e. Questa procedura conduce a gruppi indipendenti di correnti, che hanno o un carattere definito sotto parità o un carattere di chiralità definito. Quelle con carattere chirale definito sono fisicamente accettabili ed implicano che i leptoni con elicità positiva trasformano sotto SU_3 in maniere contragradienti rispetto ai leptoni con elicità negativa. Le proprietà di trasformazione permettono di assegnare numeri quantici simili alla stranezza ed allo spin isotopico. Può venire formulata una corrispondenza barioni-leptoni che fa corrispondere i leptoni ad elicità positiva a p, n e Z^- (barione con $S=3$) e i leptoni ad elicità negativa ad X^+ (barione con $S=1$), Ξ^0 e Ξ^-. La lagrangiana debole tra quattro leptoni viene scritta in una forma invariante sotto trasformazioni di spin isotopico leptonico (SU_2) e vengono esaminate le consequenze fisiche di questa invarianza.